Tutorial 5
Advanced Graph Theory

August 19, 2013

1. Count the number of spanning trees in the following three graphs using the recurrence relation learned:

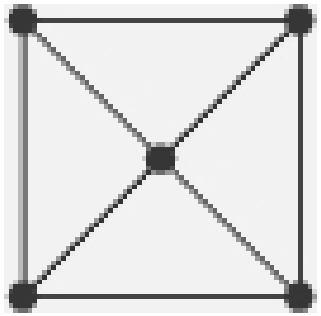

Figure: Graph 1

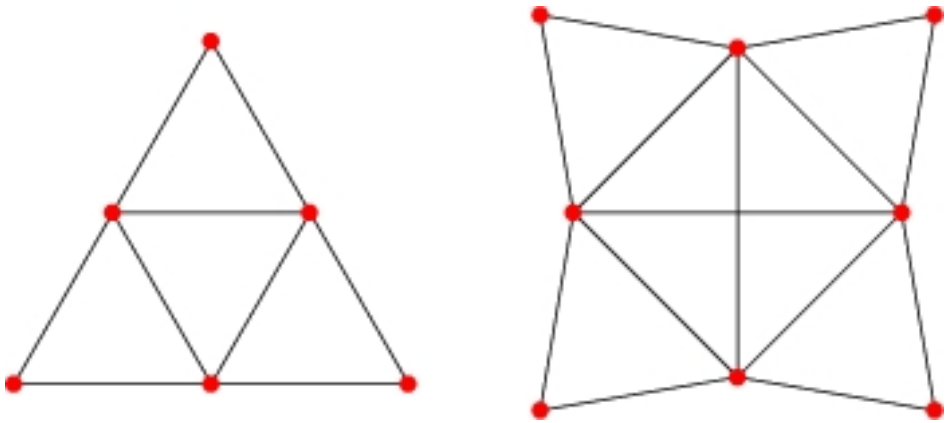

Figure: Graphs 2 and 3

2. Prove that, every n-vertex simple graph with no (r+1)-clique has at most $(1 - \frac{1}{r})\frac{n^2}{2}$ edges.
3. The Turan graph $T_{n,r}$ is the complete r-partite graph with b partite sets of size (a+1) and (r-b) partite sets of size a, where $a = \lfloor \frac{n}{r} \rfloor$ and $b = n - ra$.
   Prove that,
   - 3.1 $e(T_{n,r}) = (1 - \frac{1}{r})\frac{n^2}{2} - b\frac{(r-b)}{2r}$
   - 3.2 $e(T_{n,r}) = \binom{r}{2} + (n - r)(r - 1) + e(T_{n-r,r})$

4. Let $S(m, r)$ denote the number of partitions of an m-element set into r non-empty subsets. In terms of these numbers, count the number of trees with the vertex set $\{v_1, ..., v_n\}$ that have exactly k leaves. [Rényi [1959]]